

Mathematical meaning construction in language, tables, and images: The potential utility of language proficiency development in numeracy teaching

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Abstract

Numeracy entails a facility for communicating mathematical knowledge in diverse contexts. Indeed, large scale standardised assessments have demonstrated that numeracy is associated with both mathematics and language proficiency. And yet, while research in mathematics education in schools has demonstrated the utility of taking a discursive view of mathematics, similar developments in higher education studies are scarce. Moreover, as teaching interventions in numeracy may lead to tensions between mathematical and disciplinary knowledge, the prospect of including language proficiency development is challenging. Taking a discursive multimodal view, this study reports on a pilot teaching intervention that introduced explicit language proficiency development into numeracy teaching activities at a prominent South African university. The study found that the participants produced coherent descriptions of statistics when mathematical concepts were related to vocabulary and grammar. These observations suggest that language proficiency development has the potential to realise the goal of teaching mathematical knowledge within disciplinary curricula.

Keywords: language proficiency development, multimodal discourse analysis, numeracy, systemic functional linguistics

Introduction

Numeracy is the ability to use mathematics in modern life. This includes understanding mathematical meaning as it is expressed in language, symbols, and images. Looking at largescale standardised tests that seek to measure it as a competency, one finds formal definitions of numeracy that emphasise comprehension and communication. For instance, the framework of the Programme for International Assessment of Adult Competencies (PIAAC) defines it as 'the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life' (PIAAC

Numeracy Expert Group, 2009: 21). Similar emphasis is given to communication in the cognate concepts of mathematical literacy and quantitative literacy as defined by the Programme for International Student Assessment (PISA), and the National Benchmark Test in South Africa (NBT) respectively (Tout & Gal, 2015: 693-95; Vera Frith &Prince, 2009). It comes as little surprise, therefore, to find studies in numeracy literature that show that proficiency at both mathematical literacy and quantitative literacy are associated with language proficiency (Ercikan, et al., 2015; Prince & Frith, 2020; Grisay, et al., 2009). Given that only a small proportion of prospective students are deemed proficient at quantitative literacy by higher education standards (Frith & Prince, 2016; Scott, et al., 2007), the NBT results point to a need to incorporate academic numeracy teaching into mainstream disciplinary curricula at higher education. Yet, while the value of language proficiency for numeracy development has recently been noted, the literature already indicates that conceptions of numeracy as socially relevant lead to tensions between privileging academic mathematical knowledge and privileging disciplinary knowledge, each at the expense of the other, unless there is a weakening of the classification of traditional curricula, which may threaten the salience of the intervention itself (Frith, 2012; Frith, et al., 2010; Jablonka, 2003; Jablonka & Geliert, 2012).

Research in mathematics education indicates that early school mathematics achievement itself can be developed by integrating language proficiency development into mathematics teaching (Dröse & Prediger, 2019; Prediger, et al., 2018; Prediger & Şahin-Gür, 2019; Prediger & Wessel, 2013). This evidence suggests that mathematical tasks require elaborate language comprehension in terms of textualisation, contextualisation, and discourse. However, research that concerns incorporating these insights into numeracy teaching at higher education is scarce in numeracy literature, even though disciplinary contextualisation is central to teaching numeracy. The present study aims to demonstrate the need for, and effects of, providing explicit language proficiency development in numeracy teaching at undergraduate level. Taking a Systemic Functional Multimodal Discourse Analysis as a theoretical framework, it is a pilot intervention study about how to teach students to describe statistics given in tables and charts by guiding them through clause construction. While the intervention indicates that language proficiency development in isolation is insufficient, students who participated in the intervention were more likely to reconstruct coherent mathematical meaning in their written descriptions. This points to a need for developing students' proficiency at reading and constructing mathematical meaning in multiple modes, i.e., visual, tabular, and written, as well as the interrelations between them. In what follows, I introduce SF-MDA as a theory of meaning construction that can be applied to composite representations of language and image in documents, I describe the teaching intervention, in which students worked through worksheets designed as a guide to writing descriptions of statistics, I give an overview of the design of the research study and a summary of the results, and finally, in the discussion and conclusion, and I suggest how the observations made here may inform the teaching of Numeracy and contextualised mathematical understanding in general.

Systemic Functional Multimodal Discourse Analysis

In the work of Prediger and others, concerning the integration of language proficiency development into mathematics teaching at primary school (e.g., Prediger & Wessel, 2013), a social-semiotic notion of the linguistic register of mathematics (Halliday, 1978: 111) is combined with Duval's cognitive-semiotic notion of semiotic representation across modes (Duval, 2006). Even though this research does not take a strictly social-semiotic view of how mathematical meaning is constructed across modes, it is a good example of how mathematics can be taught as discursive and embodied practice (Gutiérrez, et al., 2010). However, this perspective has not been widely adopted in numeracy literature. In this study I have taken a social-semiotic and linguistically informed theory of multimodal discourse called Systemic Functional Multimodal Analysis (SF-MDA), as coined by O'Halloran (2007). This perspective applies the Systemic Functional Linguistics (SFL) model of language to all modes of meaning (Bartlett & O'Grady, 2017; Halliday, 1978; Halliday & Matthiessen, 2013). SF-MDA approaches investigate how texts (monomodal or multimodal) in general are articulated to show their appropriateness for their context. Studies in this tradition include investigations into multimodal documents (Baldry & Thibault, 2006; Lemke, 1998; Royce, 2007), literature (Thibault, 1991), music (van Leeuwen, 1999), visual art (O'Toole, 1994), and inter-semiosis of mathematical images and symbolism (O'Halloran, 1999a, 1999b, 2005, 2007). Moreover, the work of Kress and van Leeuwen (2006) has been important and influential. This work applies to visual design and provides an explicit 'grammar' for the kinds of meanings found in visual artefacts constructed through systems of choice according to three distinct but interrelated domains, the 'metafunctions' of SFL: representational relations, interpersonal relations, and compositional relations. However, since these studies do not give a comprehensive account of how meaning is constructed across modes in a medium, and since mathematical practices are inherently multimodal, the framework adopted here is the General Multimodal model (GeM), developed by John Bateman and his colleagues and (Bateman, 2008, 2017; Bateman, et al., 2017). This theory of documents accounts for meaning making both within modes and across them (Bateman, 2008: 41).

In multimodal linguistics, a semiotic mode is a socially and culturally shaped resource of meaning making (Bezemer & Kress, 2008: 171). Examples of semiotic modes used in teaching include speech, writing, images, and mathematical symbolism. With each mode, people – researchers, editors, teachers, and students – produce signs in accordance with their socially situated interests. Semiotic modes have three strata (Bateman, et al., 2017: 113–117). The first is the canvas, the material substrate which can be physically manipulated to produce distinctions in form that can be perceived with the senses, e.g., ink on paper, sound, computer screens. The second stratum is the semiotic resource, the system of signs expressed in the canvas that are recognised through comprehensible rules of composition. These vary in complexity from simple, at the purely lexical end (e.g., traffic lights), to the complex, at the grammatical end (e.g., written and spoken language, which have interrelated systems of choice and syntax). The final stratum is the discourse semantics, the relationship of meaning between signs as they are used and the ends which they effect in a social context (Bateman, et al., 2017: 117–121). With this nuanced

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view of mode, we can recognise written language and speech as distinct semiotic modes with different canvases and semiotic resources, e.g., ink on paper versus sound, and graphology versus phonology respectively, even though they have very similar discourse semantics, since writing evolved from spoken language (Bateman, 2011: 23–25).

Semiotic modes are embedded in mediums, which are artefacts of communication that have become a historically stabilised site for the deployment and distribution of a selection of semiotic modes for achieving various communicative goals. They are often a product of considerable craftmanship habituated to historical and situated communicative practices. A book is an example of medium, since it is traditionally used to mobilise written text, typography, page layout, and so on. They provide the material required by the canvases of their participating semiotic modes. For this reason, the material of a medium necessarily constrains the semiotic modes it can accommodate to some degree. Written language, static images, typography, page layout, and the modes associated with print have come to be depicted on new canvases given by the technology of modern smart phones and tablets. And while migration between media transforms a semiotic mode to some degree, such that the canvas and its affordances change, the discourse semantics may be carried over intact. The medium of interest here is the page, which may be depicted on material paper or on a digital screen (Bateman, 2008: 263).

The prominent semiotic mode of the page in this study is written language. In addition to the stratification of written language into canvas (phonology or graphology), semantic resources (lexicon and grammar), and discourse semantics according to the GeM model, the SF-MDA model of this mode offers two more dimensions: instantiation, and the metafunctions mentioned above. Pioneered by Michael Halliday and his colleagues, this systemic and functional model provides an elaborate theory of verbal or written language (Halliday, 1978). This model posits three metafunctions: domains of tightly intra-connected, but loosely interconnected, systems, which are defined by three corresponding aspects of the context: ideational domain of field (representational), interpersonal of tenor (interpersonal), and textual of mode (compositional) (Bartlett & O'Grady, 2017; Halliday & Matthiessen, 2013: 30). As per Halliday's dictum, 'language is as it is because of the functions it has evolved to serve in people's lives'(Halliday, 1978, p. 4). It is not surprising, therefore, that this model of language is relevant to research in mathematics education. Following the work of Pimm (1987), who observed that mathematics is a language in that it is a system for meaning making, there has been substantial interest in how language is used in different ways in the learning of mathematics as compared to everyday use (Schleppegrell, 2010). Furthermore, researchers in multimodal linguistics have deepened our understanding of the linguistic challenges of mathematics education by describing the grammatical patterns through which mathematical language is constructed (Lemke, 2003; O'Halloran, 1998, 1999b, 2003; Veel, 1999). Mathematics brings together symbolic representation, visual images, and language in ways specific to its practice (Lemke, 2003: 229; O'Halloran, 1999b). It uses a technical vocabulary and a grammatical structure that is elliptical and associated with long, dense noun phrases, frequent being and having verbs, and implicit logical relations (Lemke, 2003; Veel, 1999).

While this study is concerned with how students use written language to describe statistics presented in tables and charts, this concern is not limited to the movement of semantic content across modes, i.e., *transduction*, (Bezemer & Kress, 2008: 175), but also how meaning construction is distributed across modes. Tables are artifacts of multimodal documents, and they carry meaning through the composite semiotic mode of page-flow, which can combine elements in any of the semiotic modes appearing on the canvas of the page, including written language, typography, diagrams, graphs and so on (Bateman, 2008: 174-176). In the tables we consider here, the basic elements are derived from written language and mathematical symbolism, expressed through the semantic resources of typography, whose spatial composition is subject to a layout structure informed by Gestalt laws of perception, an area structure of a grid, and a navigational structure (Bateman, 2008; Bateman, et al., 2017: 264). Thus, any table can be decomposed into modules of text in 'text-flow'¹) according to a grid model with straight lines to demarcate rows and columns. These modules are the main entries, which are complemented by row and column headings, and title. Tables extend the semantic reach of written language by condensing relational meaning into a relatively small visual space and enables visual comparisons and the identification of patterns (Bateman, 2008: 100; Lemke, 1998: 96-101). Through linguistic ellipsis, it condenses nominal groups (usually the long noun phrases of technical discourse) to

single elements (often numeric) in a systematic way, while also using layout resources of pageflow to supply the information required (i.e., the headings) to reconstruct the original clause (Baldry & Thibault, 2006: 64–65). Nevertheless, tables typically have a high degree of 'experiential under-specification' (Baldry & Thibault, 2006: 71), and they are often designed to be read alongside an accompanying text to retrieve the full nominal groups and their underlying semantic relations (Lemke, 1998: 96). On the part of the reader, however, the potential to retrieve the intended meanings depends on the one hand, on their language proficiency as far as they are familiar with the grammatical constructions of scientific or technical discourse (Baldry & Thibault, 2006: 71; Lemke, 1998: 96; Thibault, 2011), and on the other hand, on their degree of familiarity with layout design of tables (Wright & Fox, 1970).

Charts are multimodal artifacts that are realised by the diagrammatic mode, which may be understood to be schematic images as page-flow with the addition of diagrammatic elements such as labels and connecting lines (Bateman, et al., 2017: 2701–271, 281). In addition to realising independent 'self-standing' diagrams, such as graphs, the diagrammatic mode readily combines with the resources of photography, cartography, illustrations, and written language with elements of labelling (Bateman et al., 2017: 279–294). It also draws on the resources of layout space, inviting instant interpretation according to Gestalt laws, but to a greater extent than when page-flow realises tables. Self-standing graphs are of particular interest to this study. Kress and Leeuwen have argued that the visual structures of representation in all images are either narrative, a representation of unfolding actions and events, or conceptual, generalised and timeless relations

 1 Any model of layout built around written language whose structure unfolds linear manner and may include, although non-textual elements may be embedded in the structure (Bateman, et al., 2017: 270– 271).

between participants, of which graphs are examples (Kress & Van Leeuwen, 2006: 71). They are classified as representing topographical analytical relations between quantities (Kress & Van Leeuwen, 2006: 100-101). They are analytical as they represent part-whole relationships, and they are topographical as their graphic elements are drawn to a scale that is based on the quantity or frequency of aggregates of participants that are taken to be identical. And yet in the diagrammatic mode, the communicative purpose of the chart does not depend on the graph element alone, but on the diagrammatic elements as well, including the axis labels and chart title, which are themselves realised using written language.

The teaching intervention

The participants of the teaching intervention were undergraduate humanities students at a prominent university in South Africa who were taking a one-semester quantitative research methods course in 2022. The course is targeted at, but not limited to, students studying psychology who require academic support as indicated by their performance in the National Benchmark Test for Quantitative Literacy or by their membership of a four-year extended degree foundation program. The national test indicates that a large majority of prospective students are likely to need academic support to cope with demands of QL at higher education (Prince & Frith, 2017). The course itself and the extended degree program are part of efforts by the university to address the articulation gap between secondary school and university and promote an equity of outcomes in the graduation profile, which remains skewed towards historical lines of socialeconomic inequality (Council on Higher Education, 2022). Nevertheless, extended degree programs nationwide target only a minority of students and they are in general insufficient to meet the articulation gap between secondary school and university (Scott, 2018).

Teaching on the course has led me to suspect that students' quantitative literacy is governed by their language proficiency. The objectives of the course include teaching reading and understanding statistics given in the results of contemporary quantitative studies in social science research. However, notwithstanding the learning activities of comprehension-type questions (e.g., Frith, 2012), their descriptions of these statistics tend to be incoherent with respect to context. For example, consider Table 1, which was adapted from the results of a study of the prevalence of recent drug use among adult arrestees in three cities of South Africa (Parry et al., 2004).

	Cape Town	Durban	Johannesburg			
	$(N = 335)$	$(N = 343)$	$(N = 320)$			
Cannabis	50.2	42.6	39.2			
Mandrax	31.7	21.0	19.4			
Cocaine	3.4	6.3	4.9			
Amphetamines	0.0	0.9	0.4			
Benzodizepines	127	0.2	4.4			

Table 1: Percentage of all cases that tested positive for recent drug use, by drug and site

As a tutorial exercise in facilitated in small groups, students were presented with this table along with two descriptive paragraphs taken from the original article and asked eight questions to demonstrate their understanding. The fourth question item is, "Using the context of the study, give the meaning of the value 39.2 in the column for Johannesburg." Here are the responses from two students collected during this iteration of the study:

Text 1: 39.2 of the 320 arrestees use cannabis.

Text 2: There were 39.2 cases of cannabis that tested positive in Johannesburg.

These responses were coherent with neither the mathematical figure of percentage frequency nor the context of the situation presented in the study. In Text 1, while the mathematical meaning may be restored simply adding "percent" or "%" to the numerical term, the student has given "use cannabis" as the process of action, which is not coherent with the context of the situation reported in the text. In fact, the descriptive paragraph given refers to "measures", which included "urinalysis", and the title of the table described its entries as percentages of "cases that test positive for recent drug use". Furthermore, the specific participant in the activity has not been specified, i.e., "arrestees in Johannesburg". In Text 2, no inclusion of the term "percent" or "%" would restore the mathematical meaning since the student has inserted the numerical term as an absolute count of the cases in the plural. In fact, since clauses of the exist type ("there is/are") have only one participant in the figure of experience (Halliday & Matthiessen, 2013), the grammatical choice does not facilitate expressing a relation between two participants². And with regards to the context of the situation, it is not accurate to say that some cases of cannabis tested positive, but instead that some cases tested positive for cannabis. These observations suggest that coherent reconstructions of the general context of the situation, which relates to disciplinary knowledge, and of mathematical meaning depend on students' facility with written language.

To develop the students' language proficiency at reconstructing the full context of the situation in writing, and the mathematical meaning in particular, I set up a pilot teaching intervention. Students in the course were invited to participate in intervention workshops to be coached to construct experiential meanings that were coherent with the statistics given in tables and charts in the course. The grammatical construction of experiential meaning was informed by the SFL treatment of the grammar of English (Halliday & Matthiessen, 2013). The workshops were targeted at existing question items in classroom tutorials that task students to describe the

 $^{\circ}$ Text 2 may be the student's attempt at writing "39.2 cases tested positive for cannabis in Johannesburg" in the equative form, introduced below.

meaning of statistics in context. Alongside these question items, students were given supplementary exercises as part of the intervention, in which they were to follow three steps to construct a descriptive clause in writing (Figure 1). At the first step, the students are tasked with identifying the elements of the research activity related to the statistic including participants, process, range, as well as sub-elements that specify an element further in noun phrases, i.e., number and subgroup (Halliday & Matthiessen, 2013). At the second step, they compose a clause with the identified elements according to a given template. At the final step, they rephrase this clause as a *thematic equative* (Halliday & Matthiessen, 2013: 92-97). Otherwise known as 'pseudo-clefts' in formal grammar, they are one of many constructions that help to build cohesive texts conveying emphasis with stylistic effect (Biber, et al., 1999), and they have been given much attention as tools for highlighting and presenting new information (Collins, 1991; Declerck, 1984). Moreover, each entry of a table is often a single element that represents a nominal group through ellipsis, and thematic equatives precisely reproduce the nominal groups that were elided to produce the table entries (Baldry & Thibault, 2006: 64-65). The students were scored and given feedback on each element required by the description, and overall feedback as well.

Step 2: Write a one-clause sentence (i.e., one verb group) by substituting the elements 1 to 7 identified above in the following expression.

Number	Part.	Subgroup	Process	Range	Place	Time
39.2% of	arrestees	prom Johannesburg	tested	positive for recent use of cannabis	in 8 police stations in SA	ın August/September 2000.

Step 3: Re-write the sentence in an equative form:

Figure 1 Template to guide the construction of clauses that describe a statistic

The study

The pilot intervention study presented in this article is a case study that aimed at answering the following research question:

How does developing students' language proficiency support their reconstruction of mathematical meaning and the field of the situation?

The study employed volunteer sampling. All the students in the course were invited to take part in the intervention study by means of an announcement to the whole course posted on the learning management system early in the second teaching semester of 2022. In addition, the first intervention workshop was conducted before one of the classes to demonstrate its potential utility. Out of a course of 258 students split into five classes, twenty-four students volunteered to attend the intervention workshops, and all but two of them were from the class that received the demonstration (See Table 2). Among the workshop attendees, half of them (12 students) reported speaking English at home, while the rest reported speaking isiXhosa (7), Afrikaans (3), isiZulu (2), or seSotho (1) at home, either alone or alongside English. However, a large majority of them attended schools where English was the medium of instruction (20 students), while the rest reported instruction in either isiXhosa (2) or Afrikaans (2) or English and isiXhosa (1). Furthermore, of the latter four students, English was not their home language either. It is worth noting, furthermore, that these two isiXhosa-medium schools were township schools, i.e., non-feepaying schools with matric pass rates lower than the national average and located in workingclass townships. By contrast, the two Afrikaans-medium schools were former model-C schools, i.e., fee-paying schools with matric pass rates above the national average and located in more affluent parts of the city.

Note that the students who spoke either English or Afrikaans at home, were generally more proficient at school mathematics and quantitative literacy than those who spoke either isiXhosa, isiZulu, or se Sotho according to the National School Certificate National Benchmark Test. Those who spoke English or Afrikaans at home have an average NBT QL score above 51%, while the latter group averaged below 46% (Table 2). And even though the average score at NSC Mathematics among isiXhosa speakers is highest among the language groups, only three students wrote NSC Mathematics at school out of ten speakers of isiXhosa, isiZulu, or Sesotho, compared with seven out of fifteen English or Afrikaans speakers. In fact, students often take mathematics literacy as a strategic alternative to mathematics at grade 10, and generally achieve higher marks in grade 12 at mathematics literacy than they would have achieved at mathematics (Lynn & Stott, 2021). In this study, for example, among the fourteen students who took mathematics literacy, the average score was 76% compared with 51.2% among students who took mathematics. In light of this, and the small number of participants, the descriptions of students who spoke either of the Nguni languages isiXhosa, isiZulu or Sesotho at home were analysed as a single group as were speakers of English or Afrikaans.

Discourse analysis methods according to Halliday's Systemic Functional Grammar (Halliday & Matthiessen, 2013), were applied to the participants' written answers. These were collected from two workshops, two class tests and one exam. In the first workshop, the students described two percentage frequencies in response to two question items taken from a classroom tutorial (Tut 1 - 1a, 2d) ahead of the first class-test, in which they described another percentage frequency (Test 1 - 1f). In the second workshop, the students described an odds ratio ahead of the second class-test (Tut 7 - 1bii), in which they described another odds ratio. A third workshop was held before the exam to give further support and feedback, but no answers were collected then. Finally, in the exam, the students described two odds ratios (Exam - 4f, 5f). These question items were part of the regular course tutorials, tests and exam set for the whole course and not specifically designed for this research activity. The students received detailed graded feedback to these answers given in the workshops and the tests. In the workshops, the students had the option to resubmit. However, some students did not resubmit their answers, while some did not attend at all. One student missed the first class-test, and two students did not qualify to sit the exam. Finally, in tests and the exam, some students did not answer the target question items, while some did not supply the correct response. Only the correct responses were analysed in this study. The rates at which students supplied the correct response across the three language groups are displayed in Table 3.

	Tut 1a	Tut1 2d	Test1 1f	Tut7 1bii	$Test2 -$ 3e	Exam 4f	Exam 5f
English or Afrikaans	81.3	75.0	93.8	43.8	50.0	62.5	81.3
Nguni	100	100	100	62.5	62.5	87.5	75
Total	87.5	83.3	95.8	50.0	54.2	70.8	79.2

Table 3 Correct responses to target question items across language groups as percentages

In accordance with the teaching intervention, the elements represented in each student response were recorded as either present or missing or incoherent with the text. For example, to describe an odds ratio taken from a published study (Sagatun, et al., 2007), question item 5f in the exam, "Explain, using the context of this study, the meaning of the odds ratio 0.71 in bold in Table 4" (Table 4), two students produced Texts 3 and 4, which have been scored for the seven elements necessary to coherently represent an odds ratio (Table 5). Furthermore, the equative form is noted.

Table 4 Proportion of adolescents with mental health problems stratified by weekly hours of physical activity

*Proportion of adolescents classified as having mental health problems.

Text 3: The odds of having mental health problems for boys who have physical activity 1-4 hours a week is 0.71 times as big as those who have physical activity of 0 hours per week.

Text 4: Boys who exercise 1-4 hours a week outside of school are 0.71 times more likely to have a lower score and better mental health than boys who do not exercise outside of school.

Element	Text 3	Score	Text 4	Score
Probability	The odds	1	likely	0
Participants	of boys	$\mathbf{1}$	Boys	1
Subgroup	who have physical activity	1	who exercise 1-4 hours a	
	1-4 hours a week		week outside of school	
Process 1	of having	$\mathbf{1}$	to have	
Range	Mental health problems	$\mathbf{1}$	a lower score and better	
			mental health	
Process ₂	is	$\mathbf{1}$	are	
Comparative	0.71 times as big as those	$\mathbf{1}$	0.71 times more \cdots than	Ω
	who have physical activity		boys who do not exercise	
	of 0 hours per week		outside of school	
Place		Ω		O
Equative		Yes		Nο

Table 5 Scores of two descriptions of an odds ratio in response to Exam 5f

In Text 3, the student has used the word "odds" coherently to indicate the measure of uncertainty as an objective numerical quantity and has been scored 1 point. In Text 4, however, the student has indicated probability the word "likely", which does not cohere well with the concept of odds (Holcomb, et al., 2001). Odds are a measure of uncertainty that compares the frequency of an event with the frequency of its opposite alternative as a ratio, while likelihood is a measure of uncertainty that measures the frequency of an event as a proportion of the total number of events. Text 3 coherently describes the ratio with the adverbial phrase "0.71 times as big as" as opposed to "0.71 times more" in Text 4. Neither text indicates the place, while question 5f did not give the dates of the study described, and so Time has not been scored. Finally, Text 3 uses the equative construction as per Step 3 of the workshops, while Text 4 does not. However, in the case of odds ratios, unlike with risk ratios, there is no concise way in English to express the meaning of an odds ratio without using the equative form. This underscores the utility of explicit language instruction for teaching numeracy since the mathematical meaning being described circumscribes the grammatical choices available to describe it.

Results

In general, coherence at describing the elements Participant, Subgroup, and Range increased between the first tutorial and the exam. A summary of the overall success at constructing the meaning of statistics in context is given in Table 6.

In the case of the workshops (Tutorials 1 and 7) only the students' first attempts have been scored, not their resubmissions. In the assessments some students did not provide the correct response to question items set in assessments, i.e., Test 1 and 2 and Exam, while some students did not sit for the assessment at all. For example, 19 students responded to question item 3e in Test 2, but only 13 were correct and analysed. It should also be noted that no Probability or Comparative elements were expected in Tut 1 and Test 1, since the statistics described in those question items were percentage frequencies, in contrast to Tut 7, Test 2, and Exam where descriptions were required. Finally, an association between coherence and language group seemed to hold early in the intervention, i.e., the group of English or Afrikaans speakers compared to Nguni speakers. For example, the odds of describing a Range element coherently at item 1a of Tut 1 among speakers of English or Afrikaans is 4.3 times as big as the same odds among Nguni speakers. Nevertheless, for every element the association is not statistically significant (Table 7). Furthermore, the frequencies are often not large enough for tests of significant association to be valid. In any case there is no strong association by the end of the intervention

However, coherence generally declined between the first tutorial and the first test. Between Tutorial 1 and Test 1, the proportion of students who coherently described the element Participant drops from over 80% to just over 60%, while coherence at Process drops from over 70% to just a little over 52%. This decline coincides with the change in the representation of the statistic from using a table in Tutorial 1 (see Table 1) to using a chart in Test 1 (Figure 2). In the test the students were asked to "describe the information given in the third bar (age interval 31-40) on the chart in context". However, in their descriptions students often interchanged the Subgroup, i.e., "cases testing positive for an illicit drug", with the Range, i.e., "aged 31-40 years", and described the statistic as the rate at which 31-40-year-olds have drug-related fatalities (Text 5). These descriptions are incoherent with the chart since they contradict the fact that the six percentages constructed graphically in the chart add up to 100%, which means that the chart represents a decomposition of one total (the number of drug-related fatalities) exhaustively into six mutually exclusive parts (the age-grouped drug-related fatalities). The students were presented with a graphical representation of a mathematical distribution, and yet, their descriptions were coherent with a graphical representation of a series of six rates, i.e., rate of drug related fatalities per age group.

Figure 2 In Test 1, students were asked to describe the column at "31-40".

Text 5: 29 is the proportion of 31-40-year-olds who had fatalities involving drug use in Pretoria, South Africa, between 2003-2012.

On the other hand, coherence at describing the elements Probability and Comparative remained low between the Tutorial 7 workshop and assessments Test 2 and Exam. Less than 50% of the students constructed either element coherently in writing, while the proportions coherent at Participant, Subgroup, Process and Range are substantially higher (Table 6). As we have seen in Text 4, incoherent descriptions of Probability expressed the measure of uncertainty in terms of likelihood instead of odds, while incoherent descriptions of the Comparative did not use the adverbial phrase "times as big as". And while some students described an odds ratio, incoherently, in terms of likelihood without the equative form, some students seemed to be trying to construct the equative form, even though they expressed odds as likelihood as well (Text 6).

Text 6: The odds of adolescent boys in the sample that partake in physical activity 1-4 hours is 0,71 times as likely as those who don't.

But careful analysis of the results suggests that the teaching intervention has the potential to improve students' language proficiency at describing odds ratios, which is in general challenging for two reasons. First, students are often required to calculate the odds ratios before they can be interpreted (e.g., 3e in Test 2 and 4f in Exam). Second, odds ratios themselves are often misinterpreted in professional practice (Holcomb, et al., 2001). In fact, while the difference between the coherent and incoherent descriptions of odds ratios were addressed explicitly in the intervention (i.e., whether 'likely' is admissible), the students' own course notes at the time made the same misinterpretation. Nevertheless, the students who attended the workshop of Tutorial 7 were more likely to give a correct response to the task of describing the meaning of an odds ratio, and among them, more likely to make a coherent construction in terms of vocabulary and grammar (Figure 3). These students had higher average coherence scores in the exam at 4f and 5f describing the elements Probability, i.e., using "odds" instead of "likely", Comparative, i.e., "as big as" instead of "more", and Equative, i.e., using the equative construction.

Figure 3 Average coherence rates of descriptions of odds ratios in exam according to clause elements and workshop attendance

Discussion

The results indicate that while developing students' language proficiency supports coherent reconstructions of context, it also reveals incoherent reconstructions of mathematical meaning that cut across language groups. In general, the participants in this study did not always reconstruct the mathematical figure of a distribution in writing when it was presented to them as a bar chart. We may call this incoherent reconstruction a misconception. In this chart, a distribution is constructed visually by an arrangement of six schematic images of bars in twodimensional space, which is an exhaustive analytical and topographical part-whole relationship of quantities (Kress & Van Leeuwen, 2006: 98–104), i.e., one tall rectangle of height 100 units decomposed completely into six short rectangles (Figure 4) just as a pie chart is a circle decomposed into sectors. And yet the students' descriptions frequently constructed the mathematical figure of rates, which may also be represented by a graph of six bars in a bar chart.

In the figure of a rate, each bar itself would represent an inexhaustive analytical and topographical part-whole relationship of quantities, i.e., six tall rectangles each of height 100 units decomposed incompletely into six short rectangles, one each respectively, and the tall rectangles removed from view (Figure 5).

Figure 4 Distribution represented by an image constructing an exhaustive analytical topographical relation of quantities

A distribution and a collection of rates cannot be distinguished in the semantic resources of images alone. The isolated graphs of six bars of Figures 4 and 5 are ultimately identical. However, the bar chart presented to the students in the test (Figure 2) was, in fact, realised using the composite diagrammatic mode (Bateman, et al., 2017). While the graph of the bar chart is a contribution of the semiotic mode of images, which forms one layer, the diagrammatic elements, i.e., chart title, axis titles, axis labels, etc. are a contribution of the diagrammatic mode, which forms an additional, superimposed layer. Each layer is active in its own sub-canvas, and these are stacked and combined to form a composite unit (Bateman, et al., 2017: 281). Thus, in the design of the bar chart, written language in the horizontal and vertical axis labels and titles, and the chart title guide the reader to distinguish between a distribution and a series of rates. SF-MDA analysis reveals, therefore, how a misconception of a distribution may be expressed in students' writing through an incoherent reading in the diagrammatic mode.

The results also indicate that students who receive explicit language instruction may not reconstruct an odds ratio in writing when it is presented to them in a table. Again, it is reasonable to call this a misconception. The analysis of their answers suggest that some participants have conflated relative odds with relative risk, even though odds and likelihood are distinct but related measures of uncertainty. This misinterpretation coincided with the participants' reluctance to deploy the equative form in their descriptions despite the feedback given in the workshops of

Figure 5 Rates represented by an image constructing six inexhaustive analytical topographical relations of quantities

the intervention. And yet, since there is no familiar and easily accessible adjective that corresponds to the noun 'odds' as there is for the noun 'probability' or 'likelihood', a short coherent description of an odds ratio is difficult to produce without the equative form. This demonstrates how limitations at skilful deployments of vocabulary and grammar, e.g., using the equative form, may correspond to limitations at recognising distinctions between mathematical concepts, e.g., relative odds and relative risks. And, as in the case of an isolated image representing a distribution, a ratio expressed strictly as a decimal number, i.e., only in the mode of mathematical symbols, is an ambiguous construction of an odds ratio. Again, supplementary information supplied by row and column headings and the title enable a precise reconstruction of the mathematical meaning. Once again, SF-MDA analysis has revealed how a misconception of mathematical meaning may be revealed in students' writing through an incoherent reading in the composite mode of tables.

It is evident, therefore, that expanding language proficiency development towards reading and composition proficiency in multiple composite modes may provide insights into numeracy development with substantial benefits in the context of higher education teaching in South Africa. In NBTs of Quantitative Literacy, it has been shown that scores on question items requiring the

interpretation of percentage frequencies in tables were largely limited to students who had studied English as a Home language and Mathematics at their NSC, i.e., skewed towards both language proficiency and their mathematics experience (Prince & Frith, 2020: 441–442). Elsewhere, analysing students' writing plays a crucial role in teaching. In Science Education studies, evaluating the analogies and metaphors used in written responses has the potential to reveal students' conceptual understanding (e.g., Hestenes, 2006; Lancor, 2014), while the most fundamental mathematical ideas can be traced to metaphorical concepts notwithstanding their grounding in the science of human cognition (Lakoff & Núñez, 2000). In Mathematics Education literature, furthermore, connecting language varieties (e.g., every day, academic, technical language) and multimodal representations (symbolic, graphical, diagrammatic, etc) is one of the recognised principles for the design of learning environments that support language learning in the classroom as a means for enhancing mathematics learning (Erath, et al., 2021). Crucially, it has the potential to defuse a recognised tension between privileging academic mathematical knowledge over disciplinary knowledge (e.g., Frith, 2012). On the one hand, teaching language proficiency facilitates reconstructions of disciplinary knowledge as context, and on the other hand, teaching multimodal compositions facilitates reconstructions of mathematical meaning. Such integrated teaching would be enabled by SF-MDA methods, since, as this study has shown, it has the capacity to make the construction of mathematical meaning across modes explicit.

Conclusion

This study has investigated how a pilot teaching intervention study supported students' language proficiency development as a method that has the potential to facilitate teaching mathematical knowledge without departing abruptly from the general context of the situation. Using templates to aid clause construction as a teaching intervention, students were guided through reconstructing the meaning of mathematical figures of statistics given in tables and charts. By analysing the grammatical and word choices made in written answers given in course assessments, the study found that the students frequently described statistics presented in charts or tables as rates instead of a distribution, and as a risk ratio instead of an odds ratio. Nevertheless, the study suggests that teaching students to recognise how choices in vocabulary and grammar correlate to distinctions in mathematical meaning, these students may be more likely to construct those meanings in writing coherently. These findings also suggest that broadening language development to multimodal composition development through SF-MDA methods has the potential to relate socially situated literacy experiences to learning mathematical concepts. And since language proficiency development activities in these interventions are faithful to texts in terms of representing the general context of the situation, they present an opportunity to teach academic mathematical knowledge alongside disciplinary knowledge without mutual compromise.

Access to teaching interventions that focus on language learning alongside the development of mathematical concepts should be made available to a wider range of students at higher education. This is especially true in South Africa, where students from various language backgrounds are underprepared for their chosen fields of study. This study has demonstrated that a teaching intervention that supports language learning can improve the coherence and accuracy of student writing in numeracy development across language groups. Nevertheless, further study that extends to explicit teaching about multimodal composition with mathematical figures is required to confirm that it would lead to improved coherence with mathematical meanings as well. Furthermore, larger cohorts would be needed to demonstrate improvements in students' writing across home language groups. Moreover, it should also be noted that the misinterpretation of odds ratios in the course teaching materials outside the pilot teaching intervention study may suggest that correcting this alone may have been adequate to achieve the observed differences in student writing. Again, further study is required to provide further confidence.

Author Biography

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